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Quantum

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Quantum Computing in Finance

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Introduction

There are two main types of stock analysis: technical and fundamental. At Promontory Investment Research, we engage in fundamental analysis. This type of investing involves determining the difference between the price at which a stock is trading and its in trinsic value determined by qualitative factors and models such as DCFs and Comps. On the other hand, quantitative analysis involves using mathematical and statistical models to look at data such as stock supply and demand movement. This paper will be focu sing on quantitative approaches, which quantum computing is a type of. Quantitative analysis can be broken down into two categories: quantitative and algorithmic trading. Quantitative trading uses a multitude of data sets to create models that determine probability of an outcome and then manually make trades. On the other hand, algorithmic trading relies on data from stock exchanges to code programs that execute trades on the traders behalf. Both of these approaches are widely used, but the heavy reliance on computational power due to large data sets or computationally expensive programs leads to physical barriers.

While quant-trading has grown quickly as computers became more powerful and accessible, there are three barriers that limit the benefits of this approach

Limits to technical analysis

Moore's Law Ending

We are nearing a physical limit to computational power described by Moore's Law. WHAT law? Moore's Law established in the 1930s describes how the number of transistors on a microchip doubles every two years, though the cost of computers is halved. This rule is important because transistors are the fundamental building blocks of computers and increasing the number of transistors leads to increasing computer speed. Unfortunately, this law is coming to an end due to physical phenomena that make up transistors, and thus, we are nearing a physical limit on computational power.

Increasingly Complex Problems and Optimization Needs

Everyday, our world is getting more complicated. With the increasing connectivity due to the internet and events happening across the globe quickly triggering consequences in the market here, modeling a company is becoming increasingly complicated. Thus, we are needing to factor more information into our models. This presents a challenge to classical computers that can only calculate models linearly. For example, some of the most "simple" optimization problems such as the Traveling Salesman can take thousands of years to run.[1]

Cryptography Concerns

Another concern for quantitative traders, but more widely the financial sector, is cryptography. Many banks use a cryptography method called public-key cryptography that is fundamentally based on factoring large prime numbers. While for classical computers it could take millions of years to crack this cryptography, a new type of computing called quantum computing could do it in a matter of minutes.[2]

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[1] The traveling salesman is a problem that asks: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" The computational complexity of this problem is O(n^4). That means, for each new city added, the computation time is raised to the fourth power.

[2] This point is important because it highlights the geopolitical factors involved with quantum computing. Due to the concern around cryptography from the government, federal support will continue to flow as each of the major international players vie for the advantage quantum computing brings. The quantum algorithm famous for this is known as "Shor's Algorithm."

These three barriers present a fundamental issue for quantitative traders since an area of physics known as quantum mechanics demands a physical limit on classical computer power. However, using properties from this same quantum mechanics, a new type of computer can be built known as a quantum computer.

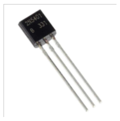


Fig 1. Transistor

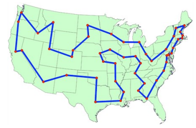


Fig 2. Traveling Salesman

Fig 3. Crytpgraphy

What are Quantum Computers?

Quantum computers are built off two main principles that allow them to overcome classical computing barriers: *superposition* and *entanglement*. Superposition is the idea that a system can be in multiple states until it is measured. Entanglement is the concept that many of these quantum superposition states can be (linearly) combined to create complex systems. Using these two principles together, quantum computers can take extremely complex problems, and instead of linearly examining each possible scenario, simultaneously sort through each possible scenario and output the "optimized" scenario.

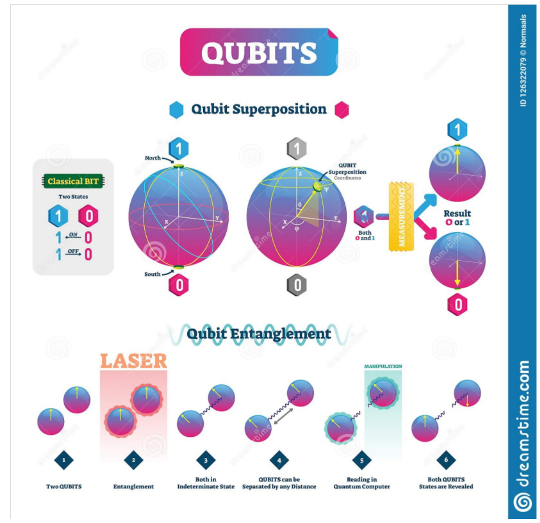


Fig 4. Qubit Diagram

How does this actually work?
To answer this question, we will quickly compare the smallest piece of hardware in each system. In classical computers, the smallest component is a bit while in quantum computers, this is called a quantum bit (Qubit for short). The difference in these components is the quality of superposition that we mentioned earlier. Classical bits are made out of transistors that do not have the quality of superposition, and thus, can only be in binary states (namely, they can be 0 or 1). On the other hand, qubits are made up of a physical system that can be in superposition (ie 0 *or* 1). This allows an large computational speed up.[3,4]

Applications

While there are substantial challenges to creating quantum computers, the transformation it could have on multiple industries has governments, companies, and scientists working overtime on projects to build the first quantum computer. In fact, the FY21 Department of Energy (DOE) has $245M for Quantum Information Science and Amazon, IBM, and Microsoft are in a race to build the first quantum computer.

So where can quantum computers even be used?

Chemistry and AI
The problem in **chemistry** is that classical computers cannot exactly simulate molecule behavior due to computation needs. In order to produce better materials or pharmaceuticals, chemists need to look at different reactions under varying conditions, but there are too many variables for a classical computer to produce a result in a reasonable amount of time.

AI is also a clear application of quantum computing because of the quantum algorithm's ability to quickly solve complex problems. Examples of areas that AI could be supported by quantum technology are search problems (such as encryption), game theory models, and decision trees.

Financial Services
Since this is a paper for a finance RSO, the financial services application will of course be the focus hereon. We can break this down into three categories of which we will focus on two.

1. Targeting and Prediction

This application focuses on customer targeting and prediction modeling where quantum computing is superior at pattern recognition, classification, and making predictions. According to IBM, 25% of small to medium financial institutions lose customers due to offerings that do not prioritize customer experience. Within

[3] (Footnote: specifically, quantum computers has processing power 2^n while classical computer have processing power 2n).
[4] So why don't we just build more quantum computers you may ask? The current challenge is to build a computer that has enough qubits to have "quantum supremacy" over a classical computer and that retain the two necessary qualities (superposition and entanglement) mentioned earlier. While these quantum properties make up the world we live in today, the "measurement" of a qubit (this can be in the form of a particle hitting it or high temperatures cause it to increase in energy) forces it to "collapse" to a "classical" state. Practically, this creates a challenge for scientists, because in the real world there are constantly particles speeding around and temperatures around 70 degrees Fahrenheit while many qubits need to be kept at -459 degrees Fahrenheit.

quantum computing, these institutions would be able to evaluate large amounts of behavioral data to target relevant products. Additionally, quantum computing can be applied to financial crash prediction.

2. Risk Profiling/Portfolio Management

A key word in finance is "risk" and how institutions are able to balance risk and hedge positions. For many banks, **Monte Carlo simulations** are the choice process to analyze risk and uncertainty. Monte Carlo simulations perform risk analysis by running models that substitute a range of values (a probability distribution) for an uncertain factor. This process is iterated over multiple times with a random set of values from the input probability functions. On a classical system, this process is usually run for 100 - 500k iterations with 100k iterations taking several hours. The **Monte Carlo simulation** is one of the applications of quantum computing we will discuss in this paper.

3. Trading Optimization

As discussed earlier, quantitative approaches to investing are very popular. One of the most known models used in quantitative finance is the **Black-Scholes model**. This model is a mathematical identity that converts an option price to an implied volatility or vice versa. At a basic level, this model is a Partial Differential Equation (PDE). This type of mathematical problem is one of the areas that quantum computers excel at. The **Black-Scholes model** is the second application of quantum computing in finance we will explore in this paper.

Monte Carlo Simulations

As mentioned earlier, quantum computers provide a large computational speed-up compared to classical computers. Monte Carlo simulations are a great application of this because of the large run-time they require. For large portfolios, this task can take overnight and sometimes multiple days on classical computers while quantum computers can reduce the time to real time or hours.

What is a Monte Carlo simulation?

A Monte Carlo simulation is a risk analysis model that inputs a chosen value in a probability distribution of an uncertain parameter and outputs the outcome of the system based on the chosen value. This process is run over thousands of iterations until we reach a probability distribition of the desired outcome.

How are Monte Carlo Simulations and portfolios related?

Since we are discussing quantitative finance, how could we leave out portfolio theory. Wait, people don't just randomly throw good stocks into the portfolio? Well, they could, but it may lead to high risk. In fact, picking stocks is so meticulous, there is a whole theory behind it, Modern Portfolio Theory. This theory outlines how to best construct a portfolio minimizing risk for a given desired return. One common way to construct an ideal portfolio is through Monte Carlo simulations. The process goes something like this:

1. Pick a list of stocks (preferably ones you want in your portfolio)
2. Obtain historical prices and calculate daily returns and standard deviation
3. Use a Monte Carlo system to *randomly* assign weights to stocks (summing to one)
4. Plot the return of each portfolio against the standard deviation (higher standard deviation, higher risk)

It is important to note that there are different uncertain inputs one can use as a parameter for a Monte Carlo simulation, but remember, that this adds a lot of runtime to the algorithm. Another item to add is that there are other metrics that are used to define the optimal portfolio (the Sharpe ratio being most common). These metrics can be used as criteria to evaluate the outputted portfolio. As mentioned earlier, it is common to run these simulations hundreds of thousands of times and thus can take many days which is extremely inefficient. This is where quantum computing comes in.

How can quantum computing help portfolio optimization Monte Carlo simulation?

There are two clear ways quantum computation can support portfolio optimization: <u>computation speed-up</u> and <u>quantum random number generation</u>.

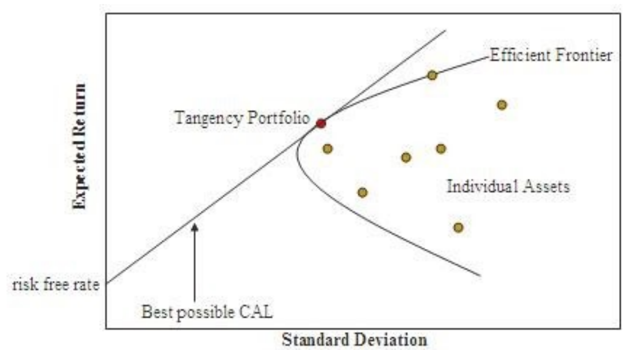


Fig. 5 Portfolio Optimization Graph

Computation speed up

The most clear way quantum computers can support portfolio optimization is by increasing the computation speed. An article from IBM states that to improve the estimate's precision by an order of magnitude on a classical computer, the number of samples in a Monte Carlo simulation would need to increase by a factor of 100, but for the same precision improvement on a quantum computer, the number of samples would only need to increase by a factor of 10. This is a clear example of the quadratic speedup quantum computing offers. Additionally, the speed up provided would allow portfolio managers to add additional parameters in and not have to worry about the run time.

Random Number Generation
While this was not emphasized above, Monte Carlo simulations are predicated on the fact that stocks are weighted *randomly*. However, true randomness cannot be coded by classical computers. (Footnote: I will not go into the details, but if you want a fun approach to think about why, you may check out Berry's Paradox or Kolomogorov complexity. It may be helpful to think about why randomness cannot be coded because the randomness is generated by an algorithm and at some point, that algorithm will repeat its pattern). Randomness is essential in Monte Carlo simulation and "better" approximations of randomness will lead to less error-prone results. A solution is to use a quantum random number generator. Since quantum computers are based on the unpredictability of physical phenomena, they are a perfect random number generator, and thus can lead to better results for the Monte Carlo simulation.

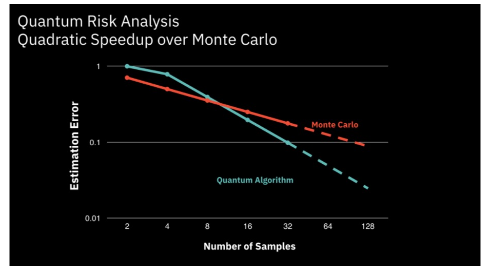


Fig. 6 Quadratic Speedup of Monte Carlo Method (IBM)

Is this helpful?
An important question to ask would be, is this actually helpful? In many ways, quantum computers would greatly help banks highly optimize their portfolios; however the main benefit would be in runtime since classical random number generators seem to be acceptably doing the job. For smaller banks who may be more risk averse and have less parameters they would like to analyze, a classical computer could fit the bill. However, for larger banks with more dollars on the line, and who want to have a slightly higher risk portfolio, investment in access to a quantum computer may be the extra edge over the market that they need. Overall the speedup time and increase in number of outputs per hour leading to a more precise and optimized portfolio are going to have to be a factor that banks will need to make moving forward.

Black-Scholes Equation

The Black-Scholes model is a mathematical equation that is used to model option pricing. Option pricing is commonly used in quantitative finance to derive the underlying price of an asset. Since by buying an option, you are betting on the future price of an asset, you quantitatively would have no idea how it may perform and could end up with a future you losing a lot of money. However, this problem is solved by the Black-Scholes model which provides present-day you more information about the future price, so that future you can hopefully gain a lot of money (or lose less money). [5]

The classical equation governs the price evolution of a call taking inputs of stock price (*S*), time (*t*), risk-free interest rate (*r*), and stock volatility (σ). This partial differential equation can be expressed as:

$$\frac{d}{dt} C(S,t) = -\frac{1}{2}\sigma^2 S^2 \frac{d^2}{dS^2} C(S,t) - rS \frac{d}{dt} C(S,t) + rC(S,t)$$

Underlying assumptions:

1. The option can be only exercised upon expiration
2. No dividends are paid out during option life
3. Markets are efficient (and therefore cannot be predicted)
4. No arbitrage possible
5. There are no transaction costs in buying the option
6. The risk-free rate and volatility are unknown
7. The returns on the underlying asset are normally distributed

The assumption that <u>markets are efficient</u> is extremely important because (depe.nding on what you may believe) markets are not always efficient. Thus, there is important information that is not included in the system. This is where quantum can come in.

Mapping the Black-Scholes model to a Quantum Problem
The Black-Scholes equation can be mapped to the Schrodinger Equation which is "THE" equation in quantum mechanics.[6] The Schrodinger Equation is a linear partial differential equation governing the time evolution of a wave function for a system which it describes. Thus, by mapping the Black-Scholes model to a free particle Schrodinger Equation, we are able to understand more information about the function.[7] The Schrodinger Equation can be expressed as:

$$i\hbar \frac{d}{dt}\psi(x,t) = [-\frac{\hbar^2}{2m}\nabla^2 + V(x,t)]\psi(x,t)$$

Where m is the mass of the particle, $\hbar$ is Planck's constant and $\psi(x,t)$ is the wavefunction. Also, the term $-\frac{\hbar^2}{2m}\nabla^2$ generally represents kinetic energy of a particle and $V(x,t)$ represents potential, or an external force applied to the particle. The Black S choles Model can be mapped to the Schrodinger Equation without the potential term (note: this is an extremely simplified derivation). The Schrodinger equation below of the free particle is also known as the heat equation so:

$$i\hbar \frac{d}{dt}\psi(x,t) = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t)$$

By mapping:

[5] I'll be discussing the European call option because it must be acted on after a certain time expiry compared to the US call options can be acted on at any time.
[6] It is relevant to note that the Black-Scholes equation is really just the heat diffusion equation implying that information in markets disperses the same way as heat does.
[7] For those who may have taken physics, you can think of this as Energy = Kinetic Energy + Potential with Potential = 0. Namely, no external force acting on the particle.

$$u = e^{\$\%\&C}(x,t), y = \log(x), s = T - t, r = \frac{1}{2}\sigma^2, z = y + \left(r - \frac{1}{2}\sigma^{!}C\ \tau, \tau = s\right.$$

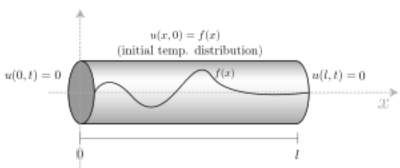
The Black Scholes Equation becomes the heat equation:

$$d\tau d\ u(\cancel{\tau, z}) = 12\,! \overset{\sigma}{-} dz\cancel{d!!u}(\tau, z)$$

This description of the Black Scholes Model is already helpful, because it can better describe an option's price, but we can also introduce back in the potential (V(x, t)) we set to 0 at the beginning of the derivation. However, we can add it back in as a time-dependent external force. By adding back in the potential, we can then again solve the Schrodinger Equation (a reinterpreted Black Scholes Equation) in the presence of market imperfections. These market imperfections are expressed as an external potential, related to the random dynamic of the underlying asset price. This allows more flexibility to the rigorous Black-Scholes Equation

Is this helpful?

Again we ask, is this actually helpful? It seems like the Black-Scholes model is very accurate and used widely, so why would we replace it with a more complicated (and potentially more confusing) version? The answer is (as it usually is) it depends! While there is much more accuracy with the Schrodinger Equation, the statistical significance of using this process depends on the input variables, and if the problem you're working on doesn't need an extremely accurate answer, it may be a better choice to use the classical Black-Scholes model. However, as mentioned in the introduction, markets are becoming increasingly more complicated, and the more information that can be implemented into the system, the better the results, and with large sums being placed on these options, firms want every edge they can get.



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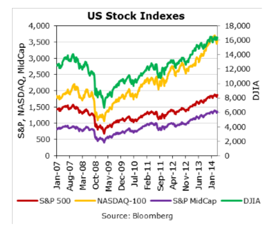

Heat dispersion can be modeled as information dispersion in markets?

The Future of Quantum Computing in Finance

Through examination of the Monte Carlo simulation and Black Scholes Model, is clear that there are applications of quantum in the finance realm. But, ***what does this mean for industry?*** As mentioned earlier, many of the most well-known banks have invested R&D into quantum computing teams. Many banks are even poaching top quantum talent from universities or quantum computing companies to head their R&D teams. While there are some major bottlenecks to quantum computing in finance such as the lack of hardware discussed earlier, it is crucial for banks to begin to look into how quantum computing can benefit them, so once these technology breakthroughs do occur, banks can jump on them early. While it may seem like the classical methods are doing "just fine," in many ways the markets are a comparison game, and if one bank has even a slight edge over another, it could cost a bank millions.

Another question you may have is, ***what will this transition look like?*** For two main reasons (technical and business), I will assert (with the minimal quantum and market knowledge an undergrad can have) that quantum computers will, initially, fit more of a "plug and play" model filling gaps in current classical processes to provide advantage.

From **the *technical perspective***, the hardware is not there to support a complete overhaul of classical technology. To contextualize, the maximum number of qubits run on a quantum computer is currently 53 qubits (on IBM's quantum computer) while the minimum number needed to run the most popular algorithms is around 500+ qubits. While this may seem like a large leap, IBM has promised a 1000-qubit computer by 2023. However, the time to disseminate this technology may take additional time past that.

From **a *business perspective***, it would be difficult to overhaul the entire financial system and replace it with hardware that is largely untested and still error-prone. Thus, switching costs are a big consideration for many firms meaning an incremental shift towards quantum computing.

Overall, while you may not run into quantum at your first day as an analyst, it may be an important topic to be aware of. While this paper has focused on its application in finance, quantum will undoubtedly reshape aspects of multiple other industries (and… may be a consideration in future DCFs?).

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